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Theoretical Evaluation of Selected Back-Calculation Methods

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ABSTRACT. Analyzing Falling Weight Deflectometer (FWD) measurements is one of the most important problems faced by transportation engineers. Most of the back-calculation programs which use these measurements, do not give unique or similar modulus values for a given pavement system. The main objectives of this research were: 1) To conduct a sensitivity analysis of a number of selected back-calculation programs, 2) To determine the ability of these programes to back-calculate assumed moduli values based on a deflection basin that was calculated using the assumed moduli values.

Six back-calculation microcomputer programs were selected and evaluated. Comparison of the results shows that CHEVDEF and ELSDEF programs are in general more reliable than other programs and can be used to back-calculate moduli without substantial errors.

1. Introduction

Pavement structural evaluation and design of overlays are inherently based on a thickness deficiency or remaining life concept which requires evaluation of the structural adequacy of the existing pavement system. There has been an increase use of nondestructive testing (NDT) of pavements for in situ material characterization,, as it can provide a quick, reliable, and effective tool to the decision makers. There are a number of testing equipments that can be used for this purpose. The Fallling Weight Deflectometer (FWD) is one of the most widely used type of these equipments.

Field studies[1-6] have indicated that deflections of the FWD correlate closely with pavement deflections induced by moving wheel loads. Consequently, layer's moduli calculated using FWD data could be regarded as good approximation of those in the field.

Several back-calculation procedures are available in a form of computer programs for back-calculating layer moduli from FWD Data.

Some analysis as well as back-calculation programs have been obtained and used to achieve the objectives of this research.

1.1 Problem Statement

The response of pavements to loading is a function of several interrelated variables. These include pavement layer properties and geometry, loading mode, and load intensity and distribution. Representing the pavement response of traffic loading by NDT techniques has become common practice in recent years. Several multilayer elastic theory computer programs are currently available to computer surface deflections of pavements if the load, layer thickness, material properties (moduli and Poisson's ratio) are known. These programs include, but not limited to, BISAR[7], and ELSYM5[8]. These programs differ in their assumptions and operating characteristics. However, in all cases, the pavement system is assumed to consist of several homogeneous, elastic and isotrophic layers. Pavement layers are assumed to be unbounded laterally with infinite subgrade thickness. Full bonding (no slip) conditions are assumed at the layer interfaces in most programs.

Determining material properties from the response of the pavement structure to surface loading (back-calculation) is not an easy task. There exists no direct theoretical solution to determine the material properties of a multilayered system from the surface deflection and the layer thicknesses. Therefore, it is necessary to perform iterative schemes in order to obtain the stiffness of various layers based on the surface deflection.

One major problem facing highway engineers today is that the available back-calculation programs for characterizing flexible pavement using deflection measurements data, do not give unique or similar modulus values for a given pavement layer.

These back-calculation programs are complex, relatively new, and still plagued by problems, including the following[9] :

1) The nonuniqueness of the resilient modulus back-calculated from the measured deflection basin,
2) Errors due to possible variation in thickness of pavement layers,
3) Errors involved in assuming a semi-infinite subgrade,
4) Time involved in the iterative process.
5) Errors in back-calculated moduli because of nonlinear behavior of granular layers and subgrade, and
6) Errors involved in using input values out of the range for which the model was calibrated.

1.2 Objectives

The specific objectives of this research are :

1) To conduct a sensitivity analysis of selected back-calculation programs.
2) To determine the ability of these programs to back-calculate theoretical moduli

values based on a deflection basin, that was calculated using the assumed theoretical moduli values.

1.3 Scope of Work

In order to satisfy the objectives of this study, a number of back-calculation programs, which are currently used to calculate the moduli of flexible pavement layers from surface deflection measurements, were selected. These programs were selected because of their availability to the researcher, and their adaption for use on microcomputer. The back-calculation programs selected are :

1) CHEVDEF
2) ELSDEF
3) FPEDD1
4) WESDEF
5) SEARCH
6) BISDEF

In order to compare these back-calculation programs, they were evaluated and the sensitivity of the calculated moduli values to variations in input parameters was investigated using standard input data.

To verify the accuracy of the selected back-calculation programs, a theoretical analysis was conducted using two analysis programs, 1) BISAR, and 2) ELSYM5, on an assumed pavement section, with known pavement layer properties, geometry, and load intensity. The estimated surface deflection measurements were then used as inputs to each back-calculation program to back-calculate the layers moduli which would be compared with the assumed values.

2. Theoretical Evaluation of Back-Calculation Programs

This section presents details of theoretical evaluation of the selected back-calculation programs. The sensitivity of the back-calculated moduli to variations in input parameters is presented. Also, the accuracy of each procedure in predicting the layers moduli is evaluated using a theoretical deflection basin calculated from assumed properties of pavement system.

2.1 Selection of Analysis Models and Back-Calculation Programs

Several multilayered linear elastic pavement models are available for use on mainframe or microcomputers to calculate stresses, strains, and displacements in pavements under different loading conditions.

Some of the multilayered linear elastic pavement models have been modified to run in a reverse-iterative fashion to determine elastic moduli from pavement surface deflections, given the layer thickness, Poisson's ratios and loading conditions. The user inputs a range of moduli for the pavement layers, and the program calculates a deflection basin. This calculated basin is compared with the measured deflection basin. The moduli resulting in the best fit between the calculated and measured deflection basin are assumed to be the correct in situ moduli for that pavement. Examples of these modified (back-calculation procedures) programs are CHEVDEF, ELSDEF, FPEDD1, WESDEF, SEARCH and BISDEF. Because these programs can run on microcomputers, they have become quite popular and are enjoying widespread use. It is possible, however, under certain conditions, to generate erroneous answers with these programs. As many combinations of moduli will result in an acceptable basin fit, the engineer must use judgement and experience to select the combination that is representative of the materials being used.

In this research two analytical models, namely BISAR and ELSYM5, were used. These programs calculate stress, strain and displacement at any point on the surface and within the pavement system given that the loading magnitude, elastic moduli, thicknesses, and Poisson's ratios of the pavement layers are known. It was found, during reviewing the literature, that these two programs are the most widely used ones, and can be adapted for use on microcomputers, such as IBM and/or IBM-compatible[10-16]. A brief description of the back-calculation and analytical programs is presented herein :

1. CHEVDEF[17] is a modulus back-calculation program, which takes measured deflections from a deflection basin with initial estimates and ranges of layer moduli and computes the modulus values that best describe the input deflection data. A linearly layered elastic computer program (CHEVRON) originally developed by Chevron Oil Company is used as a subroutine to calculate the stresses, strains and deflections. It is developed by the U.S. Army, Corps of Engineers Waterways Experiment Station (CE-WES). It can handle up to eight layers at a time.

2. ELSDEF[10,18] is a modulus back-calculation program, which uses measured deflection basin as an input. ELSDEF program is a modification of the program BISDEF. It uses ELSYM5 as a subroutine program to calculate deflections. It can handle up to five layers, and it has been developed by Brent Rauhut Engineers.

3. FPEDD1[19] is a modulus back-calculation program, which uses the measured deflection basin as an input. It can handle up to four layers. ELSYM5 program is used as a subroutine to calculate the stresses, strains, and deflections. FPEDD1 is developed by the University of Texas at Austin.

4. WESDEF[12] is a modulus back-calculation program, which uses measured deflections from a deflection basin. It can handle up to five layers. A five-layer linearly elastic computer program (WESLEA), developed by the U.S. Army, Corps of Engineers Waterways Experiment Station (CE-WES) is used as a subroutine to calculate the stresses, strains and deflections. The WESDEF program was created by combining WESLEA with an optimization routine. The optimization routine was extracted from the program BISDEF which uses BISAR as a subroutine.

5. SEARCH[20] is a modulus back-calculation program, which uses the measured deflection basin. SEARCH program uses a pattern-search technique to fit deflection basins with curves shaped like elliptic integral functions. It can handle up to three layers. This program is developed by the Texas Transportation Institute.

6. BISDEF[17,21] is a modulus back-calculation program, which uses measured deflection basin. It can handle up to four layers. BISAR program is used as a subroutine to calculate the deflections. BISDEF Program was developed by U.S. Army,

Corps of Engineers Waterways Experiment Station (CE-WES).

7. BISAR[7] is a Fortran-IV computer program for the calculation of stresses, strains, and displacements in elastic multilayer systems, induced by one or more uniform circular loads. The program is a logical extension of the earlier developed program BISTRO (Bitumen Structures in Roads) which is restricted to normal loading and in which perfect adhesion between all layers is assumed. In the BISAR (Bitumen Stress Analysis in Roads) program, the layers can be allowed to slip over each other and the loads can be a combination of unidirectional tangential and normal stresses. It is developed by the Koninklijke/Shell-Laboratorium, Amsterdam, 1979.

8. ELSYM5[8] is a five-layer elastic system responsive program. It calculates the theoretical stresses, strains, and displacements at specified points in a three-dimensional ideally elastic layered system. It is developed by Gale Ahlborn, University of California at Berkeley, U.S.A.

The above information are summarized in Table 1.

TABLE 1. A summary of back-calculation programs information.

Program	No. of layers that can be handled	Deflection and stress subroute	Developing agency
CHEVDEF	8	CHEVRON	U.S. army CE-WES
ELSDEF	5	ELSYM5	Brent Rauhat Engineers
FPEDD1	4	ELSYM5	University of Texas at Austin
WESDEF	5	WESLEA	U.S. army CE-WES
SEARCH (uses a pattern-search technique to fit deflection basin with curves shaped like elliptic integral function)	3	–	Texas Transportation Institute
BISDEF	4	BISAR	U.S. army CE-WES

All programs can be run on IBM PC computers.

2.2 Sensitivity Analysis of Selected Back-Calculated Programs

In order to investigate some of the previously mentioned problems associated with these back-calculation techniques, the sensitivity of the predicted moduli to various input parameters is evaluated. The surface deflection measurements used in performing the sensitivity analysis of each back-calculation program, were obtained from pavements of the Dhahran/Abqaiq test road. The deflection basins were obtained using the KUAB Falling Weight Deflectometer (FWD), owned and operated by the Ministry of Communications (MOC). The KUAB FWD sensors were located at 0, 12, 18, 24, 42, and 60 inches from the center of the loading plate. A typical pavement structure consisting of an asphalt concrete layer (h_1 = 3.94″ (100 mm) and an aggregate base layer (h_2 = 13.78″ (350 mm)) were used in the analysis. Figures 1 and 2 show the assumed pavement structure and the layout of the FWD sensors, respectively.

Layer 1	E_1 = 1,200,000 psi h_1 = 3.94 in.	μ_1 = 0.35
Layer 2	E_2 = 100,000 psi h_2 = 13.78 in.	μ_2 = 0.35
Layer 3	E_3 = 82,000 psi h_3 = 240.0 in. or ∞	μ_3 = 0.40
Stiff Layer	E_s = 1,000,000 psi h_s = ∞	μ_s = 0.35

FIG. 1. Pavement structure assumed for theoretical evaluation of back-calculation programs.

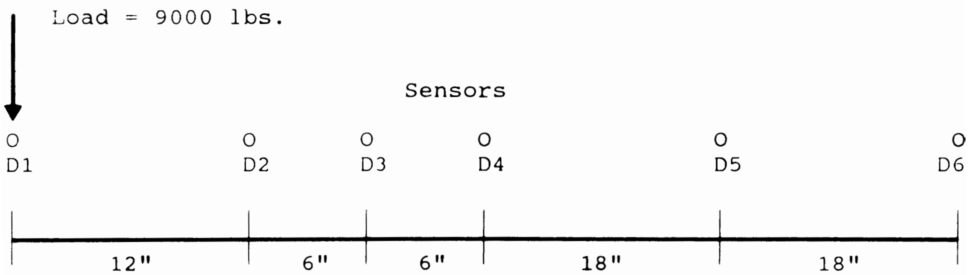

FIG. 2. Layout of FWD sensors placement.

The evaluated input parameters included all the user supplied inputs that may affect the predicted value of the layer moduli, and they are listed below :

1. Range of modulus for each unknown layer modulus
2. Depth of stiff layer
3. Initial modulus for each layer

4. Layers thicknesses
5. Magnitude of surface deflection
6. Allowable deflection match tolerances and number of iterations.

Generally, the values of these parameters area based upon the engineer experience and materials properties range. These parameters were evaluated for each back-calculation program, whenever possible. The evaluation consisted of calculating the unknown moduli of various pavement layers, using different values for each input parameter under consideration. Table 2 summarizes the assumed input values (standard) for different parameters. The following is a discussion of the results of sensitivity analysis. A flow chart showing the steps followed in the sensitivity analysis is shown in Fig. 3.

TABLE 2. Standard input data (assumed).

Variable name	Conventional asphalt concrete pavement
Modulus range (psi)	– Surfacing : 5 * E05 – 3 * E06 – Base : 3 * E04 – 2 * E05 – Subgrade : 1 * E04 – 2 * E05
Deflection values (mils) Radial dist. (in)	Sensor no. D_1 D_2 D_3 D_4 D_5 D_6 4.85 2.56 1.61 1.02 0.59 0.47 0.0 12.0 18.0 24.0 42.0 60.0
Initial modulus (seed) (psi)	– Surfacing : 7 * E05 – Base : 4 * E04 – Subgrade : 2 * E04 – Stiff layer : 1 * E06
Layer thickness (in)	– Surfacing : h_1 = 3.94 – Base : h_2 = 13.78 – Subgrade : h_3 = 240.00
Poisson's ratio (μ)	– Surfacing : μ_1 = 0.35 – Base : μ_2 = 0.35 – Subgrade : μ_3 = 0.40 – Stiff layer : μ_4 = 0.35
Number of iterations	4
Deflection match tolerance (%)	0.10

2.3 Effect of Modulus Range on Predicted Moduli

For a given pavement system, there is a combination of moduli ranges, initial moduli, and deflection basin that produce the best fit between the measured and calculated deflection basins. Results show that the best deflection match occurred mostly at modulus ranges, that were in the same order of magnitude as common modulus values of pavement layers. Table 3 illustrates an example of effect of modulus range on predicted moduli, using CHEVDEF program. Tables for other programs can be

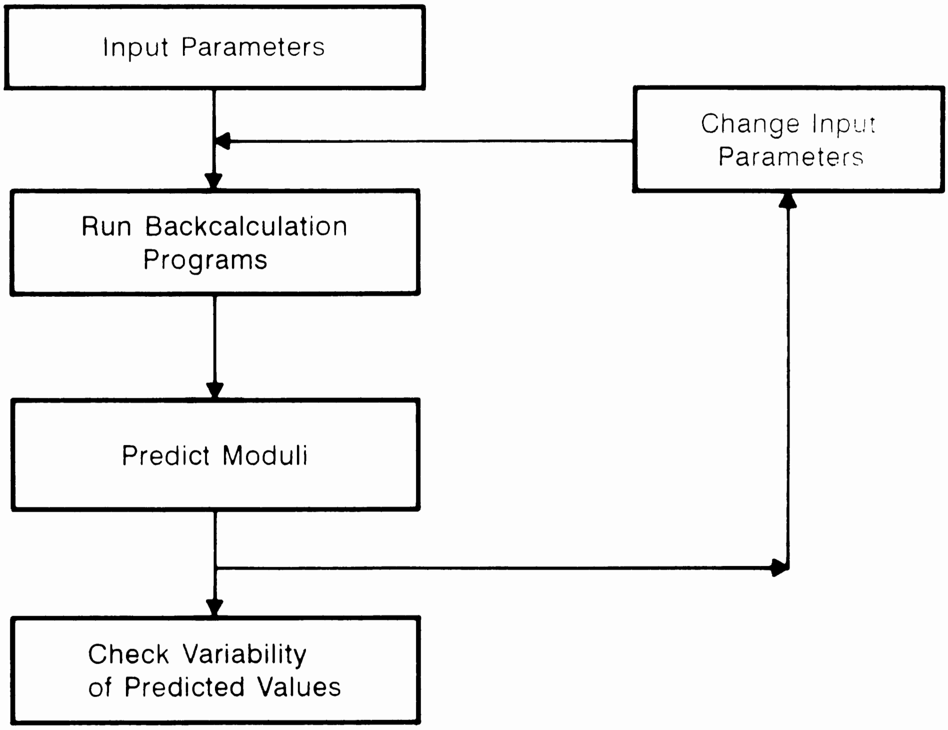

FIG. 3. Flow chart showing sensitivity analysis methodology.

found elsewhere[22]. When large moduli ranges were used, the deflection match was not so good in some of the back-calculation programs. However, very narrow modulus ranges appeared to restrict the predicted deflection values to the upper or lower range limits. It is worth mentioning that the different modulus ranges used in this part of the study were used to cover wide, narrow, and common ranges, and the numbers were randomly selected.

In almost all back-calculation programs, the deflection match difference was below 10 percent except with FPEDD1 program. This difference implies that for practical application the modulus range does not significantly affect the values of the back-calculated moduli. Otherwise, an impractical modulus range adopted can affect the predicted moduli appreciably. This modulus range input parameter applies for all back-calculation programs except SEARCH. The modulus range is supposed to improve the speed of convergence to a solution by limiting the range size in which the search for a modulus is to be conducted. Also, a practical modulus range is important, because the predicted moduli are not unique (*i.e.*, several combinations of layer moduli can result from the same deflection basin), so the appropriate modulus ranges for each layer material will limit the predicted moduli to their approximate practical values.

TABLE 3. Effect of range on predicted moduli and deflection matching (CHEVDEF).

Range class	Pavement layer	Modulus range (psi)	Predicted moduli (psi)	Deflection match difference (%)
A	- Surfacing - Base - Subgrade	1 - 3E06 1 - 3E06 1 - 3E06	3 000 001 129 849 5 524	618.90
B	- Surfacing - Base - Subgrade	1 - 2E06 1 - 2E06 1 - 2E06	1 896 889 77 634 94 413	7.56
C	- Surfacing - Base - Subgrade	1E04 - 1.5E06 1E04 - 1.5E06 1E04 - 1.5E06	1 500 000 87 521 89 458	6.35
D	- Surfacing - Base - Subgrade	5E04 - 1.8E06 5E04 - 1.8E06 5E04 - 1.8E06	1 723 058 82 646 86 529	6.02
E	- Surfacing - Base - Subgrade	1E06 - 2E06 5E04 - 1E05 4E04 - 1E05	1 664 170 83 999 86 281	6.05
F Standard	- Surfacing - Base - Subgrade	5E05 - 3E06 3E04 - 2E05 1E04 - 2E05	1 687 916 83 452 86 380	6.04

2.4 Effect of Depth of Stiff Layer

Using an infinite subgrade layer in analyzing a pavement system tends to give larger calculated deflection values than the measured. Bush[17] states that to compensate for this effect, a stiff layer should be placed at a depth of about 240 inches (6100 mm) below the subgrade surface. In this study, the position of the stiff layer was varied to determine its effect, if any, on the predicted moduli.

An example of the effect of stiff layer is shown in Table 4 using CHEVDEF program. Results for other program can be found elsewhere[22]. A graphical presentation of the effect of stiff layer is shown in Fig. 4 for the CHEVDEF program. The deflection match differences did not vary significantly when the depth of stiff layer is 240.0 inches or more for the entire back-calculation programs, except FPEDD1 program. However, the predicted moduli of surface, base and subgrade layers have varied substantially from the one predicted at the standard depth (240 inches) by as much as 25 percent at a depth of 100 inches (2540 mm). The stiff layer depth input parameter applies for all back-calculation programs except SEARCH. Results show that CHEVDEF and ELSDEF programs are identical. Theoretically, stiff layer is required to limit the depth of summation of vertical strains. If the strains are summed to infinity, the resulting calculated deflections are usually higher than the measured values. Results generally demonstrate the fact that there is an optimum stiff layer depth for a given pavement system. Therefore, the position of the stiff layer will vary as a function of the deflection-measuring device, type of pavement structure, and the adapted back-calculation program.

TABLE 4. Effect of depth of stiff layer on predicted moduli (CHEVDEF).

Position of stiff layer (in)	Predicted moduli (psi)			Deflection match difference (%)	Variation from Standard (%)		
	Surfacing	Base	Subgrade		Surfacing	Base	Subgrade
50	928 689	136 429	43 343	11.55	− 45.0	+ 63.5	− 49.8
100	1 256 852	102 302	66 464	8.12	− 25.5	+ 22.6	− 23.1
150	1 520 077	89 754	77 463	5.71	− 9.9	+ 7.6	− 10.3
240	1 687 916	83 452	86 380	6.04	0.0	0.0	0.0
260	1 691 388	83 142	87 462	5.78	+ 0.2	− 0.4	+ 1.3
300	1 682 053	82 949	89 160	5.33	− 0.3	− 0.6	+ 3.2
400	1 629 131	83 273	91 933	4.69	− 3.5	− 0.2	+ 6.4
500	1 590 609	83 264	93 836	4.57	− 5.8	− 0.2	+ 8.6
550	1 580 413	83 033	94 643	4.61	− 6.4	− 0.5	+ 9.6
600	1 575 960	82 647	95 409	4.67	− 6.6	− 1.0	+ 10.5
None	1 676 229	74 700	103 473	4.24	− 0.7	− 10.5	+ 19.8

Note: Standard depth.

2.5 Effect of Initial Moduli

The initial (seed) moduli input parameter applies for all back-calculation programs, except SEARCH, because it is an interactive type program. Table 5 presents the effect of initial moduli of surface, base, and subgrade on predicted moduli, using CHEVDEF program. Results for other programs can be found elsewhere[22]. The value of the initial modulus of all layers had minor effect on the predicted moduli of CHEVDEF and ELSDEF programs. However, results of FPEDD1, WESDEF and BISDEF programs show that these programs are more sensitive to the initial moduli parameter. Therefore, to overcome this dilemma, it is recommended that the initial moduli chosen for each layer must be within the common moduli range of the material of layer under consideration.

Overall, the initial moduli are supposed to improve the accuracy and speed of convergence to a solution by assuming practical initial value for each layer of pavement system, and also to produce the best fit between the measured and calculated deflection basins, *i.e.*, to minimize the deflection match difference. Results show that CHEVDEF and ELSDEF programs are more likely to be unique in their moduli prediction, but other programs show that several combinations of layer moduli can result from different initial moduli.

2.6 Effect of Layers' Thicknesses

The layers' thicknesses input parameter is required for all back-calculation programs considered in this research. Table 6 illustrates an example of the effect of layers' thicknesses on predicted moduli, using CHEVDEF program. Results for other programs are presented elsewhere[22]. Figure 5 is the graphical presentation of the results

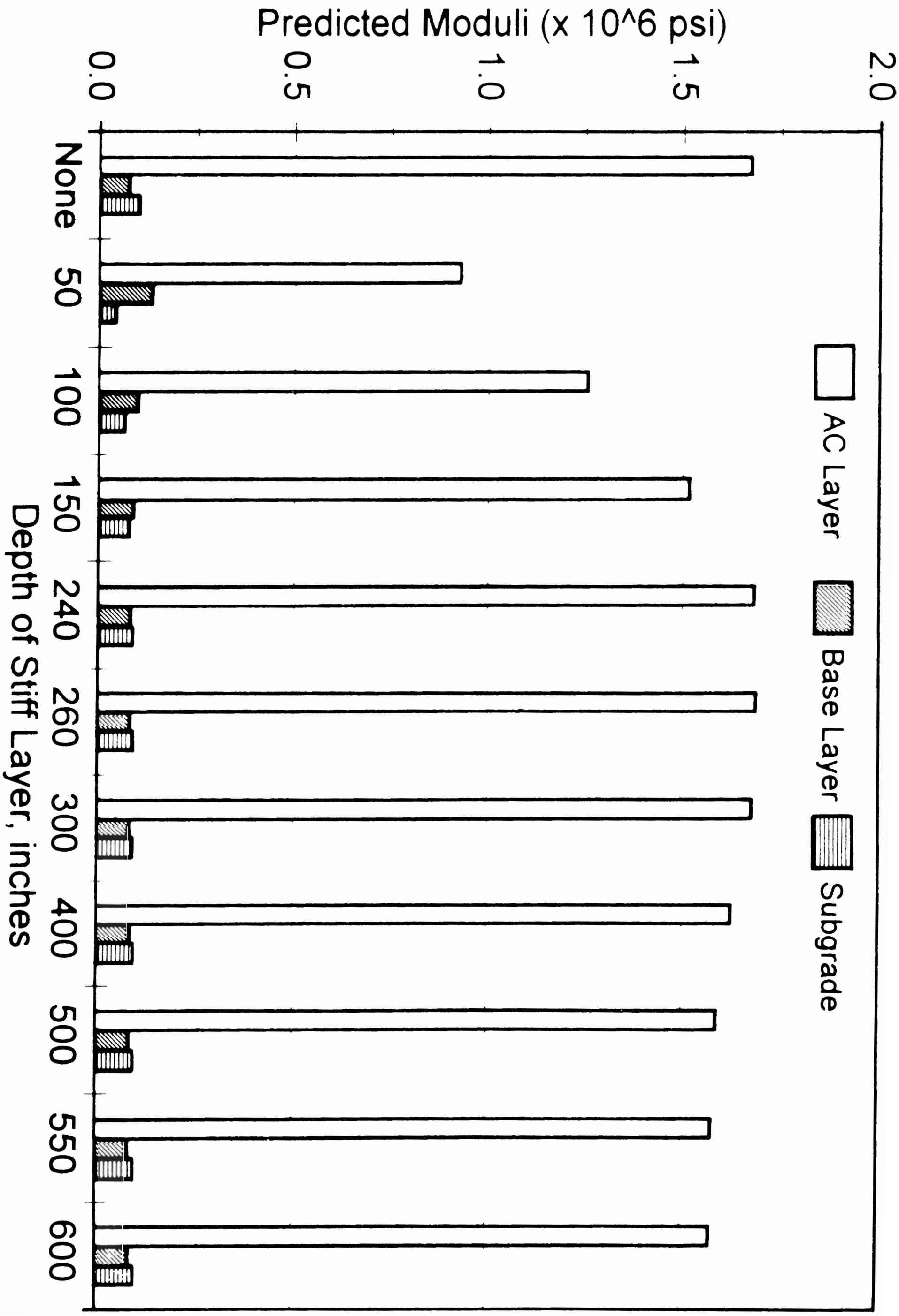

FIG. 4. Effect of stiff layer on Layer's moduli for CHEVDET.

of the sensitivity analysis for the CHEVDEF program. Generally, the predicted moduli are more sensitive to variation in thickness of surface layer than the base layer, and the layers' thicknesses have little effect on the predicted subgrade modulus. It was found that the predicted surface moduli are very sensitive to the thickness of the surface layer used, and also the predicted base moduli are sensitive to the thickness of the base layer. Moduli predicted by CHEVDEF, ELSDEF, and BISDEF are less sensitive to variation in thickness of base layer than the other programs. To eliminate the effect of this input parameter, pavement layers' thicknesses have to be accurately measured, *i.e.*, by cores taken from the pavement or by any other means.

TABLE 5. Effect of initial modulus on predicted moduli (CHEVDEF).

Position of stiff layer (in)	Predicted moduli (psi)			Deflection match difference (%)	Variation from Standard (%)		
	Surfacing	Base	Subgrade		Surfacing	Base	Subgrade
Surfacing							
5 * E05	1 694 887	83 315	86 395	6.03	+ 0.4	− 0.2	+ 0.0
6 * E05	1 690 471	83 396	86 388	6.03	+ 0.2	− 0.1	+ 0.0
7 * E05	1 687 916	83 452	86 380	6.04	0.0	0.0	0.0
1 * E06	1 683 087	83 569	86 358	6.04	− 0.3	− 0.1	− 0.0
1.5 * E06	1 641 733	84 810	86 014	6.03	− 2.7	− 1.6	− 0.4
Base							
3 * E04	1 689 942	83 424	86 383	6.03	+ 0.1	− 0.0	+ 0.0
4 * E04	1 687 916	83 452	86 380	6.04	0.0	0.0	0.0
5 * E04	1 687 224	83 467	86 376	6.04	− 0.0	+ 0.0	− 0.0
8 * E04	1 673 689	83 961	86 264	6.03	− 0.8	+ 0.6	− 0.1
9 * E05	1 672 596	83 961	86 252	6.03	− 0.9	+ 0.6	− 0.1
Subgrade							
1 * E04	1 710 016	83 061	86 403	6.01	+ 1.3	− 0.5	+ 0.0
2 * E04	1 687 916	83 452	86 380	6.04	0.0	0.0	0.0
4* E04	1 682 369	83 580	86 358	6.04	− 0.3	+ 0.2	− 0.0
7 * E04	1 677 054	83 888	86 127	6.02	− 0.6	+ 0.5	− 0.3
9 * E04	1 678 780	83 845	86 119	6.02	− 0.5	+ 0.5	− 0.3

2.7 Effect of Variation in Deflection Measurements

The effect of variation in deflection measurements input parameter was also studied. Table 7 demonstrates an example of the effect of variation in deflection on predicted moduli, using CHEVDEF program. Results for other programs are presented elsewhere[22]. Figure 6 demonstrates graphically the results of this sensitivity analysis for CHEVDEF program. The variations in deflection measurements were +/− 5, and +/− 10 percent from standard input values. Generally, all programs are sensitive to the variation of deflection measurements. CHEVDEF, ELSDEF and SEARCH programs showed equal sensitivity to the percentage of deflection variations in the surface, base, and subgrade predicted moduli. More sensitivity of predicted moduli for surface layer than the base or subgrade layers have been noticed

TABLE 6. Effect of layer thickness on predicted moduli (CHEVDEF).

Layer thickness (inches)	Predicted moduli (psi)			Deflection match difference (%)	Variation from standard (%)		
	Surfacing	Base	Subgrade		Surfacing	Base	Subgrade
Surfacing							
2.00	3 000 001	152 014	76 930	8.55	+ 77.7	+ 82.2	- 10.9
2.50	3 000 001	120 741	80 139	6.21	+ 77.7	+ 44.7	- 7.2
3.00	3 000 001	94 291	84 294	5.71	+ 77.7	+ 13.0	- 2.4
3.94	1 687 916	83 452	86 380	6.04	0.0	0.0	0.0
4.00	1 625 013	83 203	86 424	6.06	- 3.7	- 0.3	+ 0.8
5.00	961 043	79 434	87 114	6.58	- 43.1	- 4.8	+ 0.8
Base							
12.00	1 714 122	82 439	86 365	6.07	+ 1.6	- 1.2	- 0.0
13.78	1 687 916	83 452	86 380	6.04	0.0	0.0	0.0
14.00	1 685 144	83 558	86 379	6.03	- 0.2	+ 0.1	- 0.0
15.00	1 674 820	83 957	86 386	6.00	- 0.8	+ 0.6	+ 0.0
17.00	1 658 264	84 569	86 398	5.94	- 1.8	+ 1.3	+ 0.0
18.00	1 650 939	84 835	86 386	5.91	- 2.2	+ 1.7	+ 0.0

Note: Standard moduli are E(AC) = 1,687,916, E(BASE) = 83,452, E(SUB.) = 86,380 psi.

from the results of FPEDD1 and WESDEF programs. Whereas, for BISDEF program the predicted moduli for surface and subgrade layers are more sensitive to deflection variation than that for base layer (shown in Ref. [22]).

TABLE 7. Effect of deflection measurement variation on predicted moduli (CHEVDEF).

Average variation (%)	Predicted moduli (psi)			Deflection match difference (%)	Variation from Standard (%)		
	Surfacing	Base	Subgrade		Surfacing	Base	Subgrade
+ 10	1 529 555	75 853	78 393	6.18	- 9.4	- 9.1	- 9.2
+ 5	1 610 846	79 464	82 201	6.12	- 4.6	- 4.8	- 4.8
0	1 687 916	83 452	86 380	6.04	0.0	0.0	0.0
- 5	1 771 749	87 893	90 999	5.94	+ 5.0	+ 5.3	+ 5.3
- 10	1 864 423	92 826	96 143	5.84	+ 10.5	+ 11.2	+ 11.3

Note: Standard moduli are E(AC) = 1,687,916, E(BASE) = 83,452, E(SUB.) = 86,380 psi.

2.8 Effect of Tolerance and Number of Iterations

The effect of tolerance and number of iterations was applied to CHEVDEF, ELSDEF, and FPEDD1 programs only, as for others they are not applicable. Table 8 demonstrates an example of the effect of tolerance and number of iterations on predicted moduli, using CHEVDEF program. Other tables are presented in Ref.[22]. The results of sensitivity analysis using these three programs, are presented in Fig. 7 to 9. The predicted moduli of CHEVDEF and ELSDEF are insensitive to percentage deflection tolerance, but sensitive to the number of iterations when it is less than 4. When the percentage deflection tolerance is less than 0.20 percent, the predicted moduli by FPEDD1 program is insensitive, but for tolerance levels greater than or

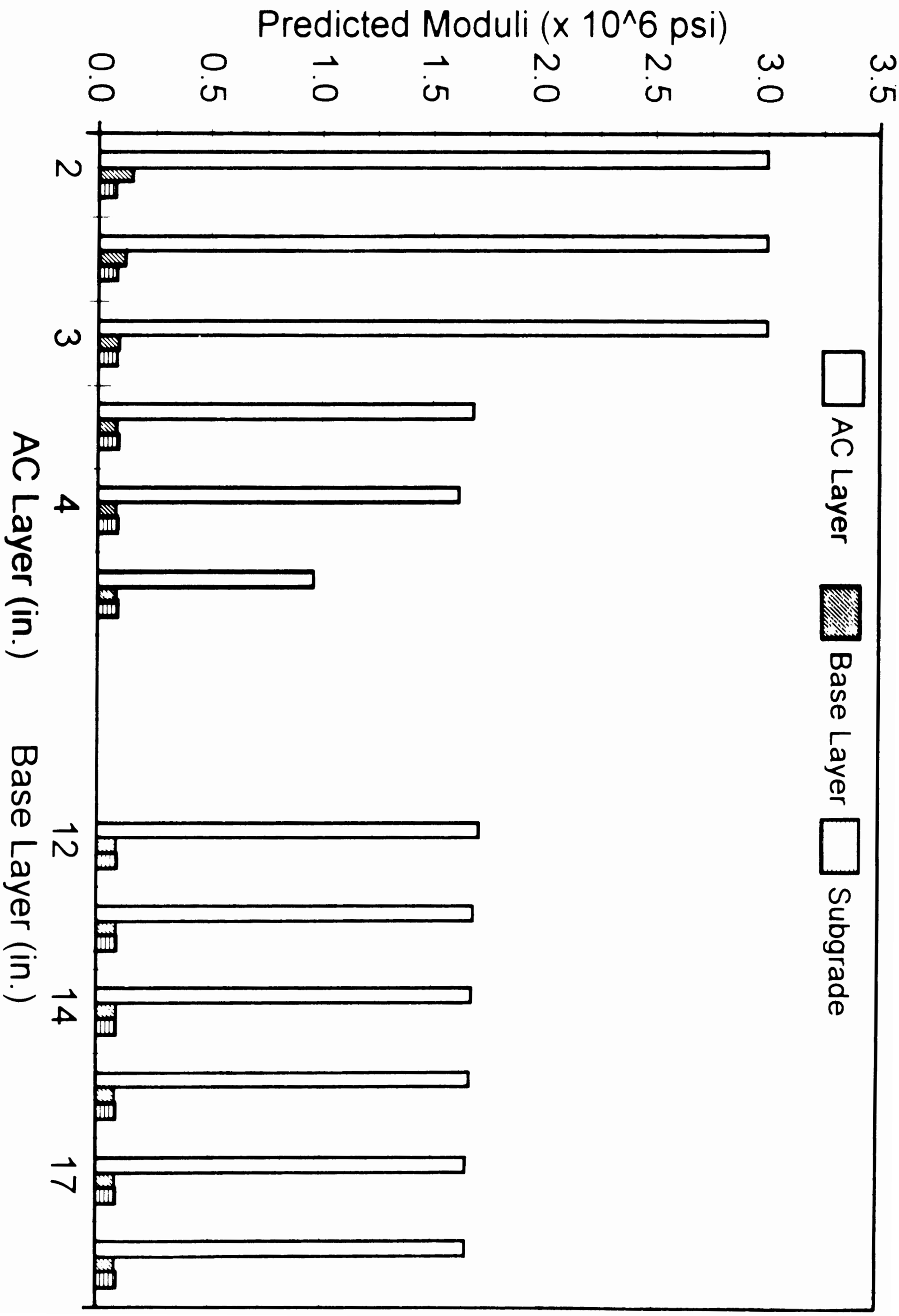

FIG. 5. Effect of layer thickness on Layers' moduli for CHEVDEF.

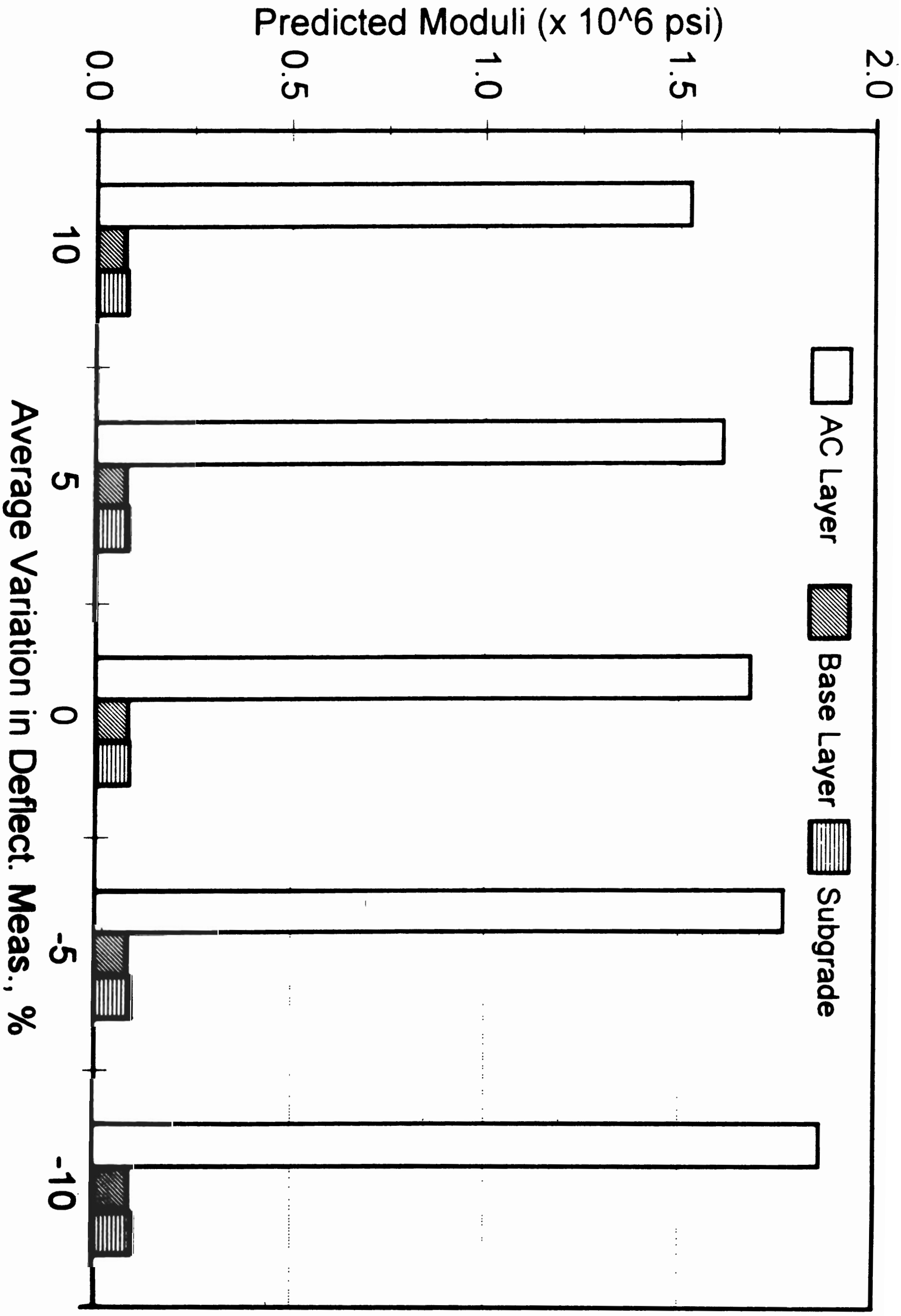

FIG. 6. Effect of deflection measurements variation on Layers' moduli for CHEVDEF.

equal to 5 percent, the predicted modulus of base layer changes appreciably especially for higher values of tolerances. The surface, base, and subgrade predicted moduli by FPEDD1 program were sensitive to the number of iterations, as it requires about 10 iterations to reach a stable levels of moduli (shown in Ref. [22]).

TABLE 8. Effect of tolerance and number of iteration on predicted moduli (CHEVDEF).

Tolerance and iteration	Predicted moduli (psi)			Deflection match difference (%)	Variation from Standard (%)		
	Surfacing	Base	Subgrade		Surfacing	Base	Subgrade
Tolerance							
0.05	1 687 916	83 452	86 380	6.04	0.0	0.0	0.0
0.10	1 687 916	83 452	86 380	6.04	0.0	0.0	0.0
0.20	1 687 916	83 452	86 380	6.04	0.0	0.0	0.0
5.00	1 687 916	83 452	86 380	6.04	0.0	0.0	0.0
10.00	1 687 916	83 452	86 380	6.04	0.0	0.0	0.0
Number of iterations							
1	2 185 679	43 899	144 033	18.32	+ 29.5	- 47.4	+ 66.7
2	1 797 564	80 591	81 751	6.29	+ 6.5	- 3.4	- 5.4
3	1 613 955	86 075	85 982	6.01	- 4.4	+ 3.1	- 0.5
4	1 687 916	83 452	86 380	6.04	0.0	0.0	0.0
5	1 642 299	84 758	86 028	6.03	- 2.7	+ 1.6	- 0.4
10	1 675 984	83 916	86 140	6.02	- 0.1	+ 0.6	- 0.3
15	1 645 819	84 636	86 101	6.04	- 2.5	+ 1.4	- 0.3

Note: Standard moduli are E(AC) = 1,687,916, E(BASE) = 83,452, E(SUB >) = 86,380 psi.

In order to compare the overall sensitivity of the selected back-circulation programs to various input parameters, the average coefficient of variation (c.v.) (standard deviation/average predicted modulus value) for each program was calculated and shown in Table 9. The results show that four programs, CHEVDEF, ELSDEF, WESDEF, and BISDEF; are about the same in their sensitivity to input parameters, while SEARCH and FPEDD1 are more sensitive, FPEDD1 is being the worst. This generally supports the earlier conclusions that CHEFDEF and ELSDEF are the least sensitive programs.

TABLE 9. Overall average coefficient of variation for predicted moduli using various back-calculation programs.

Program	Average coefficient of variation (no. of layer sets)
CHEVDEF	9.4 (10)
ELSDEF	9.0 (10)
FPEDD1	21.6 (10)
WESDEF	9.6 (10)
SEARCH	12.6 (3)
BISDEF	9.6 (8)

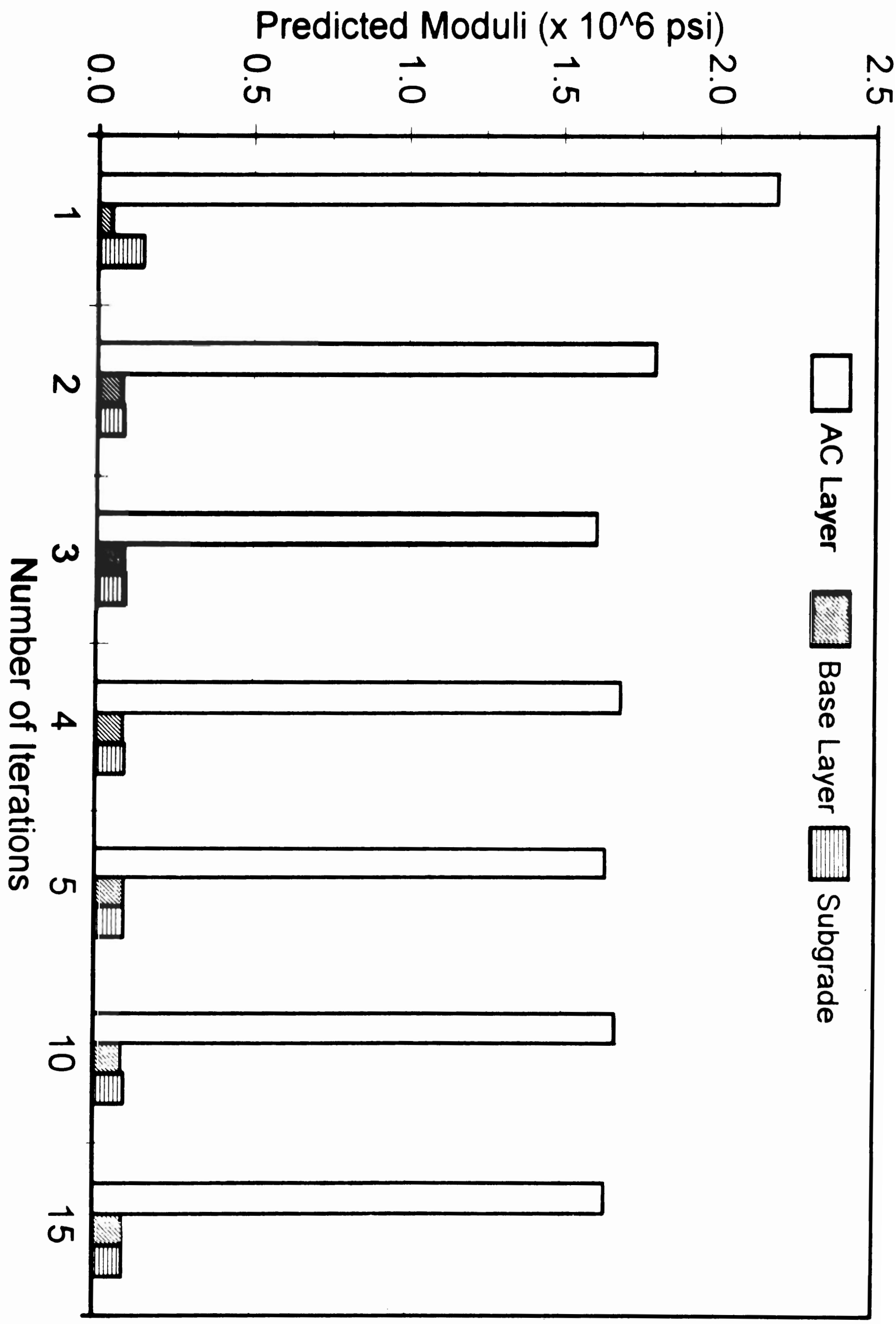

FIG. 7. Effect of number of iterations on Layer's moduli for CHEVDEF.

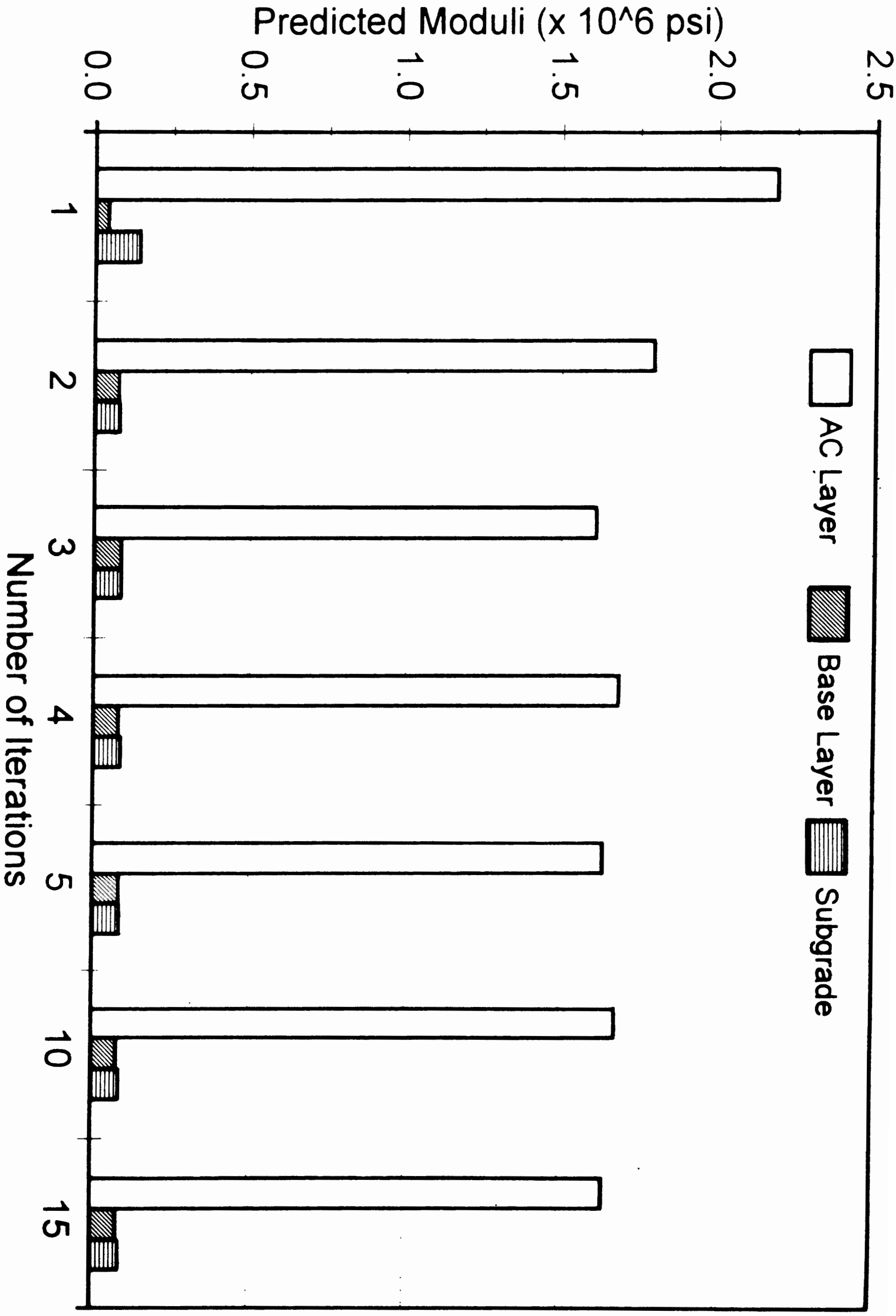

FIG. 8. Effect of number of iterations on Layer's moduli for ELSDEF.

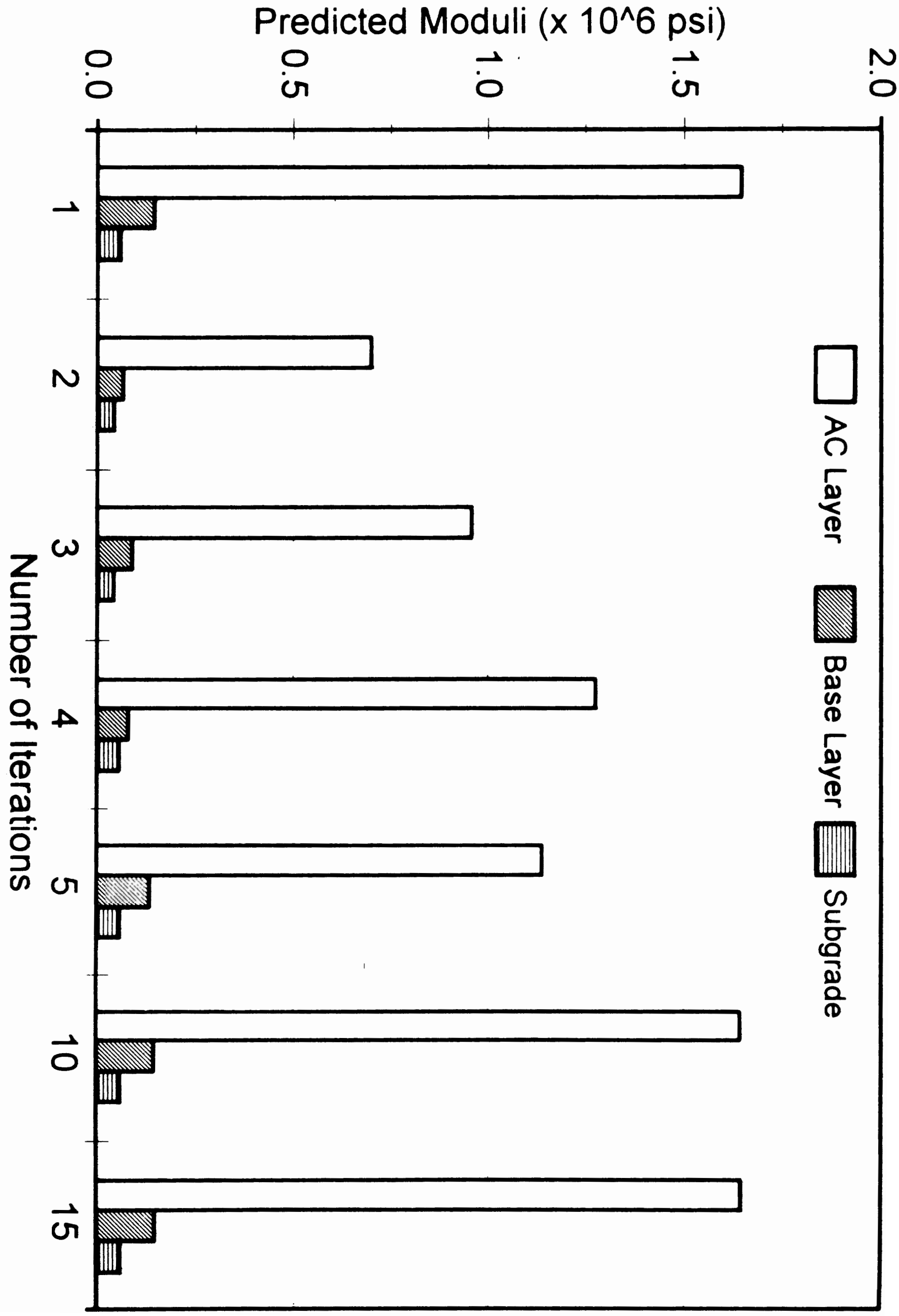

FIG. 9. Effect of number of iterations on Layer's moduli for FPEDD1.

3. Implication of Findings

The findings from the sensitivity analysis show that the six back-calculation programs have some limitations. The most important is that the predicted modulus is very sensitive to some of the user supplied inputs. Some of these inputs cannot be physically measured (*e.g.*, depth of stiff layer and initial moduli). However, to arrive at a reasonable solution from these programs, one has to be aware of these limitations and develop methods for dealing with them. Even with identical input values, the predicted moduli from different programs are different. Figure 10 demonstrates the differences in the back-calculated moduli using the standard input values presented in Table 2 for different back-calculation programs. It can be noticed that CHEVDEF and ELSDEF programs have the highest moduli results, and their results are almost the same. These two programs are followed by FPEDD1, WESDEF, SEARCH and BISDEF. The difference in results between SEARCH and other programs is probably due to the difference in stress distribution between the Vlasov and Leont'ev equation, used in this program, and the elastic layer theory used in CHEVDEF, ELSDEF, FPEDD1, WESDEF, and BISDEF programs. The other reason for this difference may be that some programs use a standard depth to the rigid (stiff) layer whereas SEARCH program does not use such a layer. Therefore, the vertical strains are summed to infinity, resulting usually in higher calculated deflections than the measured values, so the predicted moduli are generally underestimated.

The speed of computation using these back-calculation programs depends on the hardware support available, and availability of a math-coprocessor. On a relative scale, SEARCH program is the fastest, followed by ELSDEF and WESDEF programs at about the same speed, then come BISDEF and CHEVDEF programs followed by FPEDD1 program.

One major drawback of the back-calculation programs used in this analysis is their inability to consider the stress sensitivity of the predicted modulus in any given layer. Except for CHEVDEF and FPEDD1 programs, this option was not used in this research because it cannot be used as a comparison parameter between the selected programs. If the effect of stress and strain variation on predicted moduli were taken into account, there would not be a need to use a fictitious stiff layer.

Another major weakness is that the moduli determined with these back-calculation programs are never unique; there are several combinations of layer moduli that can result from the same deflection basin. This problem has been addressed by Uddin *et al.*[23] by using regression equations to determine the initial (seed) moduli. However, such approaches are often based on locally developed relationships that cannot be used with confidence outside the area for which they were calibrated.

In general, the results clearly reveal the problems that might be encountered in attempting to use most of the available back-calculation programs. A general guideline is that, before adopting any back-calculation program for moduli prediction or detailed analysis, a sensitivity analysis study should be carried out. Such study should look at all user-supplied input data, especially those that cannot be physically measured.

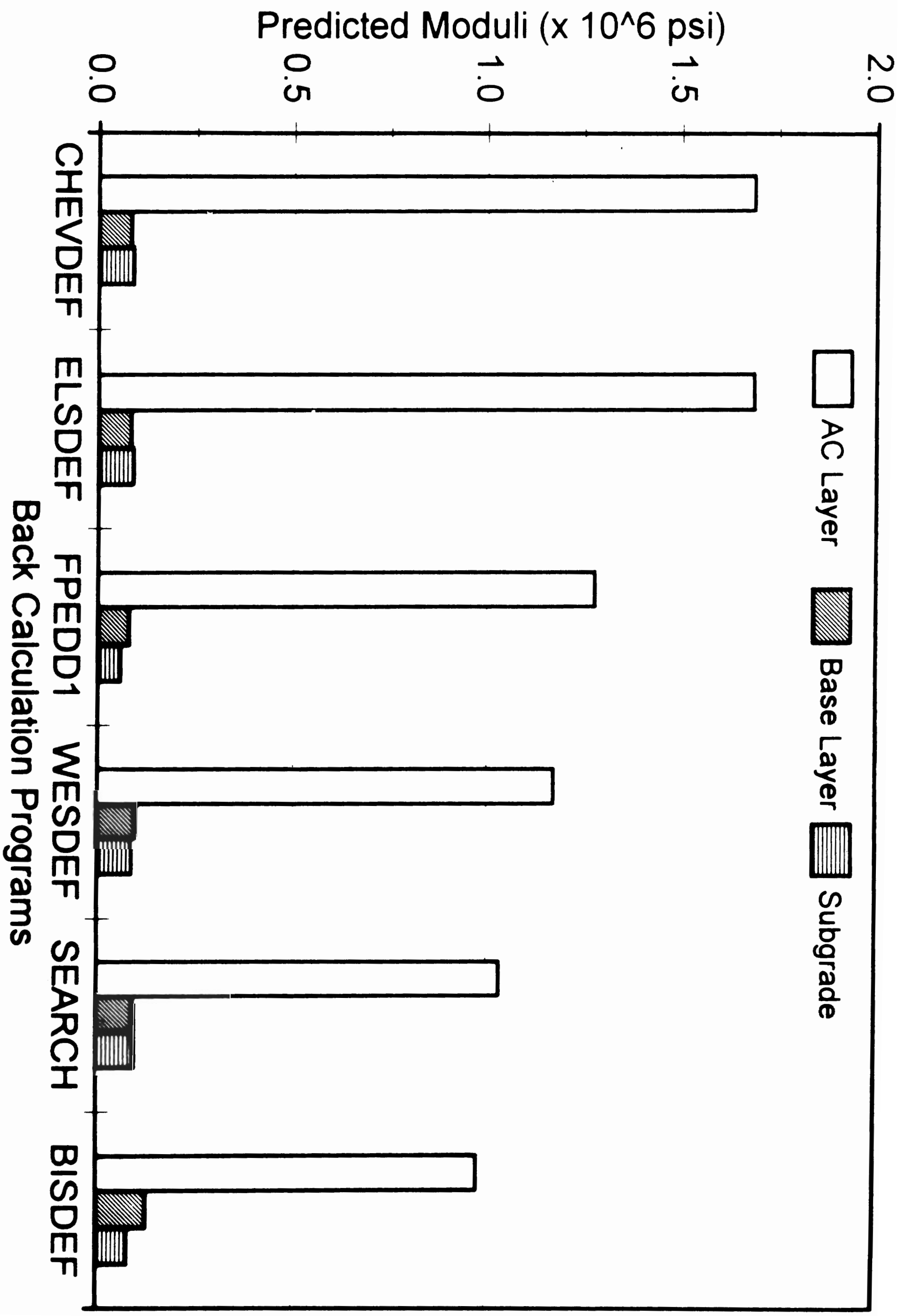

FIG. 10. Moduli of standard data.

4. Evaluation of Back-Calculation Programs Using a Theoretical Deflection Basin

The reliability of back-calculation programs studied was established by verifying the accuracy of each program under consideration. This was done by assuming a pavement system with a specific parameters (similar to test sections used in Dhahran/Abqaiq test road). Table 10 shows the input data for sample runs on analysis programs. The pavement structural parameters used in this analysis are the same as those shown in Fig. 1. Two analysis programs based on elastic layered theory were used to investigate the variation in computing the deflection basin, if any, and also to check if there is any difference in the moduli prediction of the back-calculation programs when using either one of them. The two analysis programs, namely; BISAR and ELSYM5, were used to develop theoretical deflection basins. These deflection basins were used as inputs for each back-calculation program to predict layers moduli. It should be noted here that the moduli used to develop the theoretical deflection basin were considered as actual layer moduli. Furthermore, all parameters assumed in Table 10 were held constant for the back-calculation process. The entire process is outlined in Fig. 11. Table 11 shows deflection results using BISAR program for two cases, the first case using a semi-infinite subgrade layer, the second using a stiff layer under the assumed pavement structure. The deflection basins calculated assuming a semi-infinite layer are greater than those computed assuming a stiff layer below the subgrade. The reason is that the vertical strains in the first case are summed to infinity, which resulted in higher computed deflections as explained earlier. Table 12 presents the same information, but for ELSYM5 program. The large percent differences shown in both tables, which resulted from assuming a stiff layer under the subgrade, especially at sensors D5 and D6, indicate that these sensors represent the stiffness of the subgrade layer. This result is complying with the finding of Kilareski and Anani[24], all deflection readings obtained at least 24 inches from the load center would provide reasonable estimate of the subgrade modulus. Figure 12 illustrates the percent differences between the deflection basins of both cases, semi-infinite subgrade and stiff layer, for BISAR and ELSYM5 analysis programs. The curves show that BISAR and ELSYM5 analysis programs. The curves show that BISAR is more sensitive than ELSYM5 program to the stiff layer in D_1 only. Table 13 presents the resulting output from sample runs on both analysis programs, using semi-infinite subgrade layer. It is obvious from the percent differences that both programs are producing almost the same deflection basins, as the differences never exceed 0.06 percent.

Table 14 presents output from sample runs on analysis programs, using a stiff layer at 240.0 inches (6100 mm) below the subgrade. From these results, it can be seen that the two analysis programs yield different deflection basins. At the first sensor (D_1), which is located exactly under the applied load, the maximum percent difference of 2.17% occurred. As for other sensors (D_2, D_3, D_4, D_5, D_6) the deflection percent differences decrease gradually to a value of 0.06% for D_6 sensor.

TABLE 10. Data input for sample runs on analysis programs.

Layer no.	Layer type	Thickness in (mm)	Poisson's ratio	Actual moduli (psi)
1	AC	3.94 (100)	0.35	1,200,000
2	BASE	13.78 (350)	0.35	100,000
3	SUBGRADE	240.0* or ∞ (6100 or ∞)	0.40	82,000

*When using stiff (rigid) layer below subgrade.
Loading radius = 5.91 inches
Load = 9,000,000 pounds

Sensors location = (inches)	D_1,	D_2,	D_3,	D_4,	D_5,	D_6,
	(0.0)	(12.0)	(18.0)	(24.0)	(48.0)	(60.0)

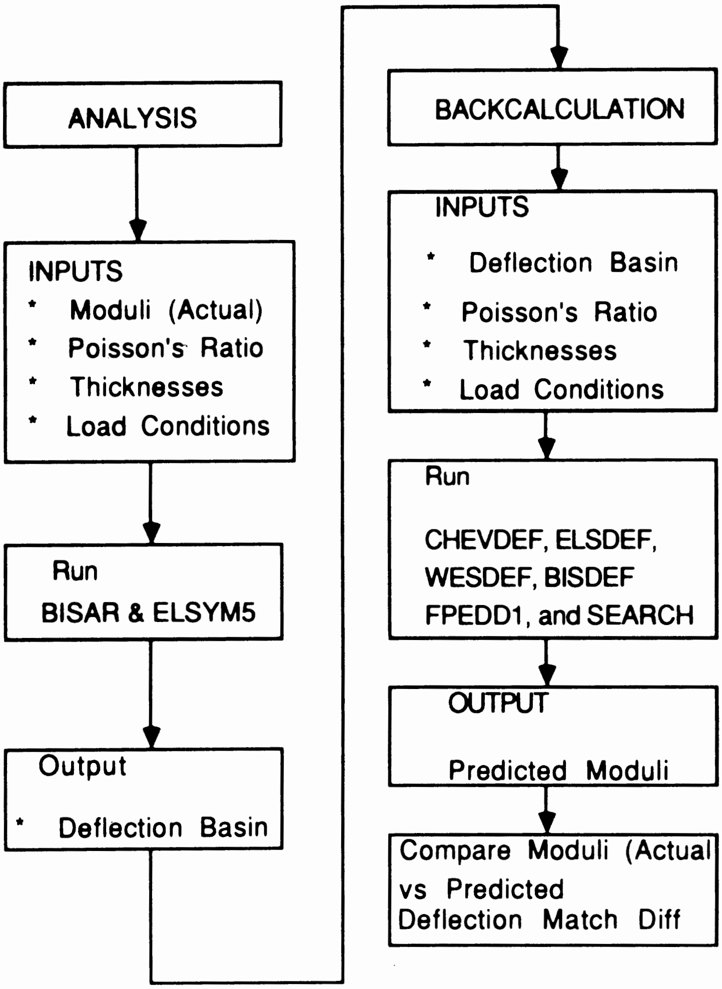

FIG. 11. Flowchart of analysis process.

TABLE 11. Resulting output from sample runs using BISAR program.

Subgrade layer state	Computed deflections (mils)					
	D_1	D_2	D_3	D_4	D_5	D_6
Semi-infinite	5.0490	2.5620	1.7390	1.2820	0.7130	0.4930
Stiff layer at 240 in.	4.8870	2.4080	1.5850	1.1290	0.5603	0.3428
% Difference	3.31	6.39	9.72	13.55	27.25	43.82

TABLE 12. Resulting output from sample runs using ELSYM5 program.

Subgrade layer state	Computed deflections (mils)					
	D_1	D_2	D_3	D_4	D_5	D_6
Semi-infinite	5.0480	2.5620	1.7400	1.2820	0.7130	0.4930
Stiff layer at 240 in.	4.9930	2.3910	1.5710	1.1250	0.5610	0.3430
% Difference	1.10	7.15	10.76	13.96	27.09	43.73

Note: 1 mil = 1 / 1000 inch = 0.0254 mm.

TABLE 13. Resulting output from sample runs on analysis programs, using semi-infinite subgrade layer.

Subgrade layer state	Computed deflections (mils)					
	D_1	D_2	D_3	D_4	D_5	D_6
Semi-infinite	5.049	2.562	1.739	1.282	0.713	0.493
Stiff layer at 240 in.	5.048	2.562	1.740	1.282	0.713	0.493
% Difference	0.02	0.00	0.06	0.00	0.00	0.00

TABLE 14. Resulting output from sample runs on analysis programs, using stiff layer at 240.0 in. below the subgrade.

Subgrade layer state	Computed deflections (mils)					
	D_1	D_2	D_3	D_4	D_5	D_6
Semi-infinite	4.8870	2.4080	1.5850	1.1290	0.5603	0.3428
Stiff layer at 240 in.	4.9930	2.3910	1.5710	1.1250	0.5610	0.3430
% Difference	2.17	0.71	0.89	0.36	0.12	0.06

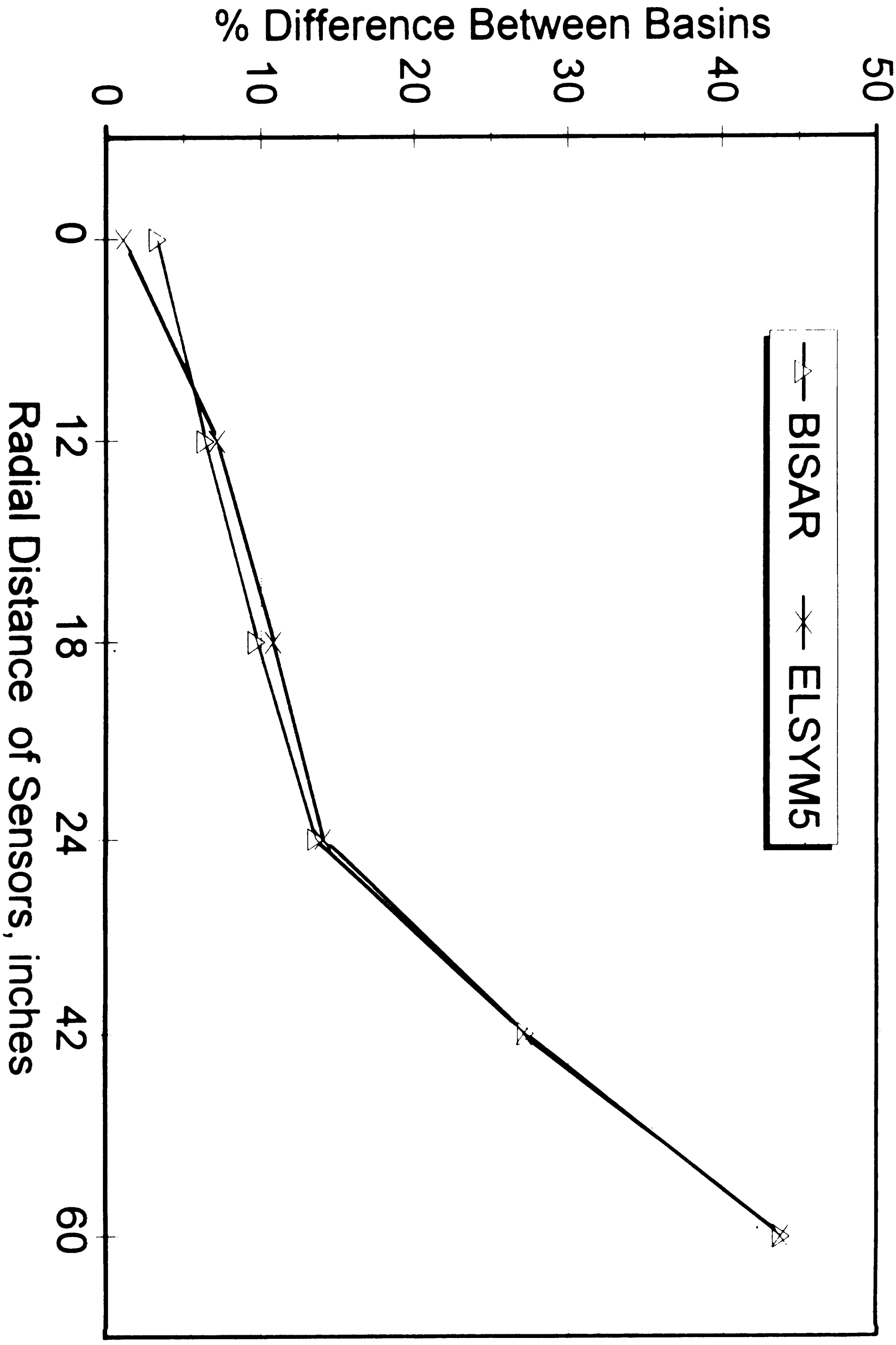

FIG. 12. Effect of stiff layer on basins.

The deflection basins generated in Table 13 were used as an input data to back-calculate the elatic moduli of pavement layers using the programs selected for this study. Table 15 shows a comparison between actual moduli and predicted ones assuming a semi-infinite subgrade layers. The results clearly show that CHEVDEF and ELSDEF programs are producing the best correspondence between predicted and assumed moduli. This is true for both analysis programs. The back-calculation programs were ranked according to their accuracy.

TABLE 15. Comparison of actual stiffness with values predicted by back-calculation programs for each of the analysis programs, using semi-infinite subgrade layer.

Program name	Back-calculated moduli (psi)			Deflection match difference (%)	Variation from Actual (%)			Accuracy ranking
	Surfacing	Base	Subgrade		Surfacing	Base	Subgrade	
By BISAR								
CHEVDEV	1 200 567	99 973	82 022	0.1	+ 0.5	- 0.3	+ 0.03	2
ELSDEF	1 199 865	100 005	82 000	0.004	+ 0.01	+ 0.1	0.00	1
FPEDD1	1 135 520	90 000	82 032	0.02	- 5.40	- 10.00	+ 0.04	4
WESDEF	1 123 219	112 924	76 612	1.70	- 6.40	+ 12.90	- 6.60	6
SEARCH	1 180 300	88 900	86 900	6.39	- 1.60	- 13.10	+ 6.00	3
BISDEF	1 138 554	112 263	77 609	1.60	- 5.10	+ 12.30	- 5.40	5
By ELSYMS								
CHEVDEV	1 201 821	99 930	82 016	0.08	+ 0.15	- 0.07	+ 0.02	1
ELSDEF	1 195 706	100 071	82 038	0.07	- 0.36	+ 0.07	+ 0.05	2
FPEDD1	1 139 344	90 000	81 552	0.02	- 5.05	- 10.00	- 0.55	5
WESDEF	1 214 988	105 549	76 988	1.30	+ 1.25	+ 5.55	- 6.11	3
SEARCH	1 180 300	86 900	86 900	0.17	- 1.64	- 13.10	+ 5.98	4
BISDEF	1 089 512	115 307	77 118	1.60	- 9.21	+ 15.31	- 5.95	6

Note: Actual moduli are E(AC) = 1,200,000, E(BASE) = 100,000 and E(SUBGRADE) = 82,000 psi.

The deflection basins presented in Table 14 were used to back-calculate the pavement layers moduli and the results are shown in Table 16. The table presents a comparison between actual moduli and the back-calculated moduli using a stiff layer below the subgrade layer. The sensitivity of the analysis programs to stiff layer is clearly indicated by the deflection match difference. BISAR program has generated higher deflection match differences than ELSYM5. This indicates that BISAR is more sensitive to the stiff layer. Generally, results of back-calculation programs are better when using output of ELSYM5 program. Comparing the results with the assumed ones reveals that CHEVDEF and ELSDEF are predicting almost the same as actual (assumed) layers moduli. The rest of the back-calculation programs are more or less predicting different moduli from the assumed ones. These results show that CHEVDEF and ELSDEF programs are the best among others. The accuracy ranking as shown in Table 16 indicates that SEARCH program is the least reliable.

TABLE 16. Comparison of actual stiffness with values predicted by back-calculation programs for each of the analysis programs, using stiff layer.

Program name	Back-calculated moduli (psi)			Deflection match difference (%)	Variation from Actual (%)			Accuracy ranking
	Surfacing	Base	Subgrade		Surfacing	Base	Subgrade	
By BISAR								
CHEVDEV	1 386 489	96 968	82 230	0.17	+ 15.5	- 3.00	+ 2.80	4
ELSDEF	1 382 453	97 190	82 142	0.12	+ 15.2	- 2.80	+ 0.20	3
FPEDD1	1 191 617	100 880	78 723	0.01	- 0.7	+ 0.90	- 4.00	1
WESDEF	1 391 595	97 202	82 597	0.80	+ 16.0	- 2.80	+ 0.70	5
SEARCH	888 200	97 400	97 400	11.30	- 26.0	- 2.60	+ 18.80	6
BISDEF	1 335 344	100 167	82 176	0.60	0.60	+ 0.80	+ 0.20	2
By ELSYMS								
CHEVDEV	1 197 357	100 094	82 124	0.01	- 0.20	+ 0.10	+ 0.20	2
ELSDEF	1 199 833	99 971	82 026	0.01	- 0.01	- 0.03	+ 0.03	1
FPEDD1	1 157 717	100 447	78 723	0.02	- 3.50	+ 0.45	- 4.00	3
WESDEF	1 074 039	105 890	81 701	1.00	- 10.50	+ 5.90	- 0.40	4
SEARCH	786 100	98 400	98 400	10.60	- 34.50	- 1.60	+ 20.00	6
BISDEF	1 045 144	81 416	81 416	1.20	- 12.90	+ 8.60	- 0.70	5

Note: Actual moduli are E(AC) = 1,200,000, E(BASE) = 100,000 and E(SUBGRADE) = 82,000 psi.

5. Conclusion

From the examination and analysis of the data, the following conclusion are drawn :

1. Pavement moduli predicted by all back-calculation programs evaluated were sensitive to impractical pavement moduli ranges.
2. The surface layer was the most sensitive to errors caused by improperly assumed rigid layer depth. Under this condition, the moduli of the surface and subgrade layers are overestimated, while the modulus of the base layer is underestimated.
3. CHEVDEF and ELSDEF programs were less sensitive than FPEDD1, WESDEF, and BISDEF programs to the initial (seed) moduli input parameter.
4. All back-calculation programs were sensitive to thickness variation.
5. Generally, all back-calculation programs evaluated in this study were sensitive to the variation of deflection magnitude.
6. CHEVDEF and ELSDEF were less sensitive to the percentage deflection tolerance and the number of iteration (4 or more), whereas the other back-calculation programs were sensitive to these two input parameter.
7. It was found that CHEVDEF and ELSDEF programs have the best accuracy among the other back-calculation programs when semi-infinite subgrade layer is used.

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تقويم نظري لعدد من الطرق المختارة للحساب العكسي لمعامل الرجوعية

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المستلخص . إن تحليل قياسات المطرقة الساقطة هي أحد أهم المشاكل التي تواجه مهندسي النقل ، فمعظم برامج الحساب العكسي التي تستخدم مثل هذه القياسات لحساب معامل الرجوعية لا تعطي قيم وحيدة أو متشابهة لمعامل الرجوعية لنفس طبقات الرصف . لذا فإن الهدف من هذا البحث هو :

١ – إجراء تحليل لحساسية عدد من البرامج المختارة لعدد من العوامل المؤثرة عليها .

٢ – تعيين مقدرة هذه البرامج على إعادة حساب معاملات رجوعية مفروضة اعتمادًا على حوض انحناء محسوب باستخدام قيم هذه المعاملات الرجوعية المفروضة .

ولغرض هذه الدراسة فقد اختير وقوم ستة برامج حاسب آلي للحساب العكسي . وقد بينت النتائج أن البرنامجين CHEVDEF ، ELSDEF هما بشكل عام الأحسن من بين البرامج الستة ، وبالإمكان استخدامهما للحساب العكسي لمعاملات الرجوعية بدون أخطاء كبيرة .